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Modeling and analysis of pHRI with Differential Game Theory

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Motivations

Why pHRI:

- i) Humans and robots have complementary skills
- ii) Some tasks require necessarily two agents (e.g., large/deformable objects co-manipulation)

Motivations and Objectives

Why pHRI:

- i) Humans and robots have complementary skills
- ii) Some tasks require necessarily two agents (e.g., large/deformable objects co-manipulation)

Objective:

- i) analyze behaviors of the human—robot dyad in a peer and leader-follower fashion
- ii) towards Role Arbitration!

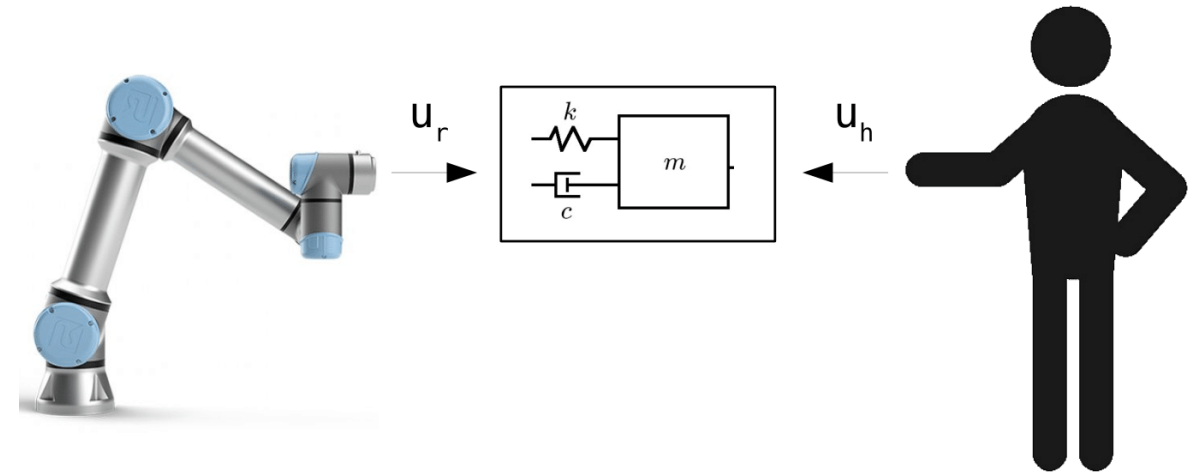
Method

- i) Modeling interaction *i.e.* we want to model how a human and a robot can possibly interact with a system, in the specific case, a Cartesian Impedance Controller
- ii) Game Theory (Differential): perfect for modeling behaviors of players interacting with a system in a rational way – differential because it models a system that evolves in time
- iii) Two GT models are analyzed cooperative and non—cooperative

The system: Cartesian Impedance

The low-level Cartesian Impedance Control intrinsically safe and compliant

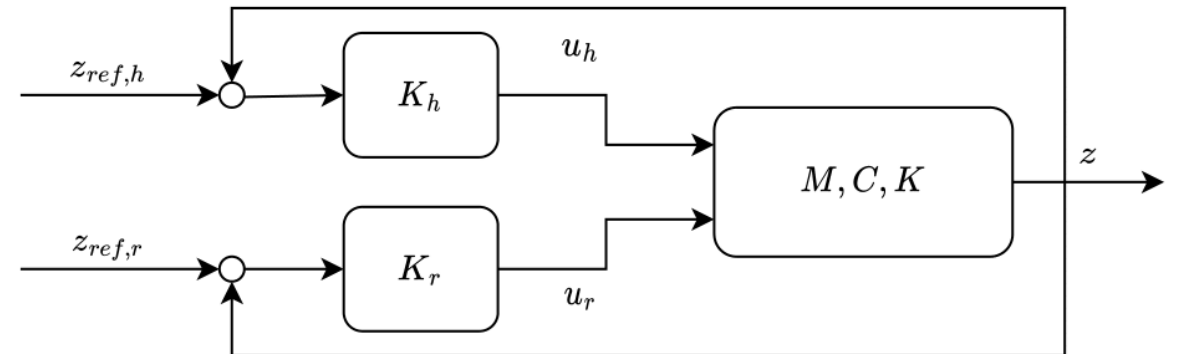
The human and the robot apply a force (measured and virtual, respectively) to move the system towards a target



Non—Cooperative GT

Human and robot may have different targets $z_{ref,h/r}$

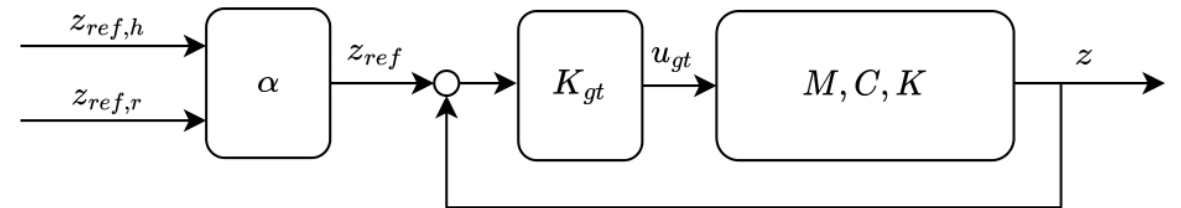
Each agent pursue its own objective



Cooperative GT

Human and robot may have different targets $z_{ref,h/r}$

The agents agree on a shared objective to improve their own return (wrt Non-Cooperative)



Objectives at comparison

LQR:

NCGT:

CGT:

Objectives at comparison

LQR:

$$J_{h,lqr} = \int (z - z_{ref,h})^T Q_h (z - z_{ref,h}) + u_h^T R_{h,h} u_h$$
$$J_{r,lqr} = \int (z - z_{ref,r})^T Q_r (z - z_{ref,r}) + u_h^T R_{r,h} u_h$$

NCGT:

CGT:

Objectives at comparison

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NCGT:

$$J_{h,nc} = \int (z - z_{ref,h})^T Q_h (z - z_{ref,h}) + u_h^T R_{h,h} u_h + u_r^T R_{h,r} u_r$$
$$J_{r,nc} = \int (z - z_{ref,r})^T Q_r (z - z_{ref,r}) + u_h^T R_{r,h} u_h + u_r^T R_{r,r} u_r$$

CGT:

Objectives at comparison

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$$J_{h,lqr} = \int (z - z_{ref,h})^T Q_h (z - z_{ref,h}) + u_h^T R_{h,h} u_h$$
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NCGT:

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$$J_{r,nc} = \int (z - z_{ref,r})^T Q_r (z - z_{ref,r}) + u_h^T R_{r,h} u_h + u_r^T R_{r,r} u_r$$

CGT:

$$J_{h,c} = \int (z - z_{ref,h})^T Q_{h,h} (z - z_{ref,h}) + (z - z_{ref,r})^T Q_{h,r} (z - z_{ref,r}) + u_h^T R_{h,h} u_h$$
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Objectives at comparison

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NCGT:

$$J_{h,nc} = \int (z - z_{ref,h})^T Q_h (z - z_{ref,h}) + u_h^T R_{h,h} u_h + u_r^T R_{h,r} u_r$$

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CGT:

$$J_{h,c} = \int (z - z_{ref,h})^T Q_{h,h} (z - z_{ref,h}) + (z - z_{ref,r})^T Q_{h,r} (z - z_{ref,r}) + u_h^T R_{h,h} u_h$$

$$J_{r,c} = \int (z - z_{ref,h})^T Q_{r,h} (z - z_{ref,h}) + (z - z_{ref,r})^T Q_{r,r} (z - z_{ref,r}) + u_r^T R_{r,r} u_r$$

By agreeing



$$J_{cgt} = \alpha J_{h,c} + (1 - \alpha) J_{r,c} = \int (z - z_{ref})^T Q_{cgt} (z - z_{ref}) + u^T R_{cgt} u$$

Objectives at comparison

$$Q_{cgt} = \alpha(Q_{h,h} + Q_{h,r}) + (1 - \alpha)(Q_{r,h} + Q_{r,r})$$

LQR:

$$R_{cgt} = \begin{bmatrix} \alpha R_h & 0 \\ 0 & (1 - \alpha) R_r \end{bmatrix}$$

NCGT:

$$z_{ref} = Q_{cgt}(z_{ref,h}Q_h + z_{ref,r}Q_r)$$

CGT:

$$J_{h,c} = \int (z - z_{ref,h})^T Q_{h,h}(z - z_{ref,h}) + (z - z_{ref,r})^T Q_{h,r}(z - z_{ref,r}) + u_h^T R_{h,h}u_h$$

$$J_{r,c} = \int (z - z_{ref,h})^T Q_{r,h}(z - z_{ref,h}) + (z - z_{ref,r})^T Q_{r,r}(z - z_{ref,r}) + u_r^T R_{r,r}u_r$$

By agreeing



$$J_{cgt} = \alpha J_{h,c} + (1 - \alpha) J_{r,c} = \int (z - z_{ref})^T Q_{cgt}(z - z_{ref}) + u^T R_{cgt}u$$

Simulations*

Case i): variations of $\alpha = \{0.2, 0.5, 0.9\}$

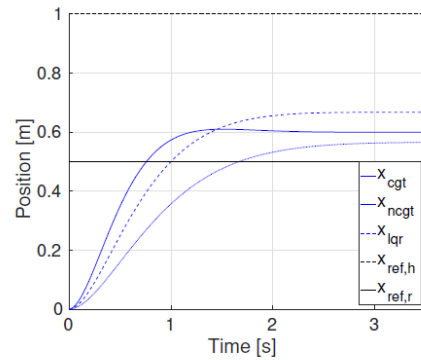
Case ii): small $Q_{r,r} = \begin{pmatrix} 0.1 & 0 \\ 0 & 0.001 \end{pmatrix}$, $\alpha = \{0.01, 0.05, 0.1, 0.2, 0.5, 0.9\}$

Case ii): small $R_{r,r} = \{5e^{-5}, 1e^{-4}, 1e^{-3}\}$

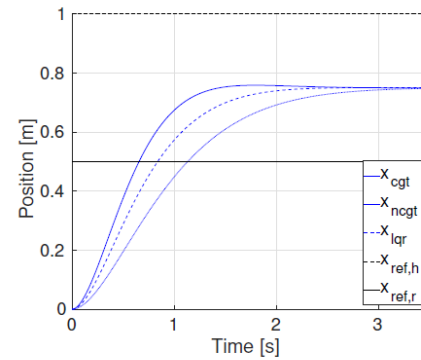
*only the robot's cost function parameters can be arbitrarily selected, the human's are recovered via Inverse Optimal Control¹

¹ P. Franceschi, N. Pedrocchi and M. Beschi, "Inverse Optimal Control for the identification of human objective: a preparatory study for physical Human-Robot Interaction," 2022 IEEE 27th International Conference on Emerging Technologies and Factory Automation (ETFA), Stuttgart, Germany, 2022, pp. 1-6, doi: 10.1109/ETFA52439.2022.9921553.

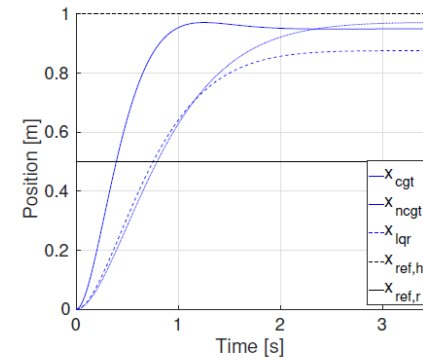
Simulations - $\alpha = \{0.2, 0.5, 0.9\}$



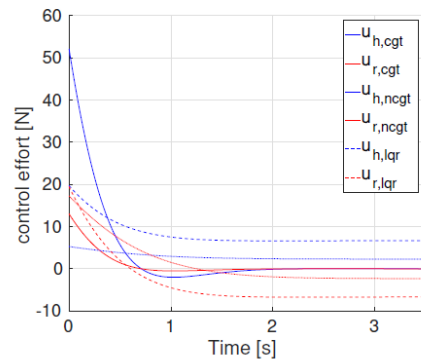
(a) shared position with $\alpha = 0.2$.



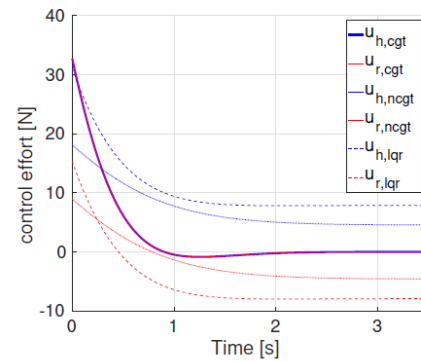
(b) shared position with $\alpha = 0.5$.



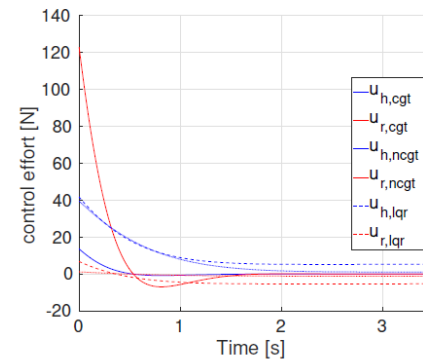
(c) shared position with $\alpha = 0.9$.



(d) control effort with $\alpha = 0.2$. Blue lines human control actions, red lines robot.

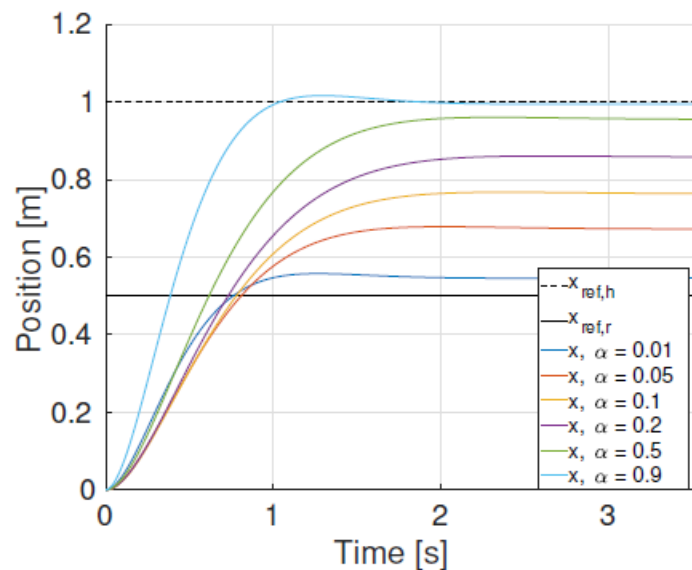


(e) control effort with $\alpha = 0.5$. Blue lines human control actions, red lines robot.

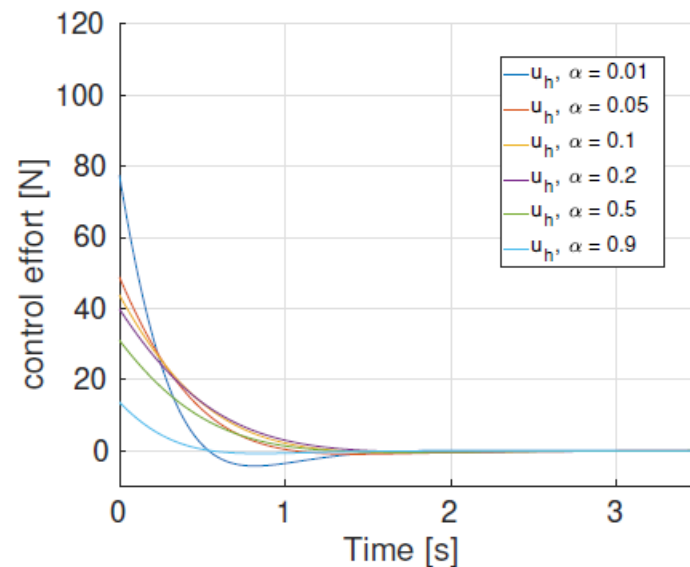


(f) control effort with $\alpha = 0.9$. Blue lines human control actions, red lines robot.

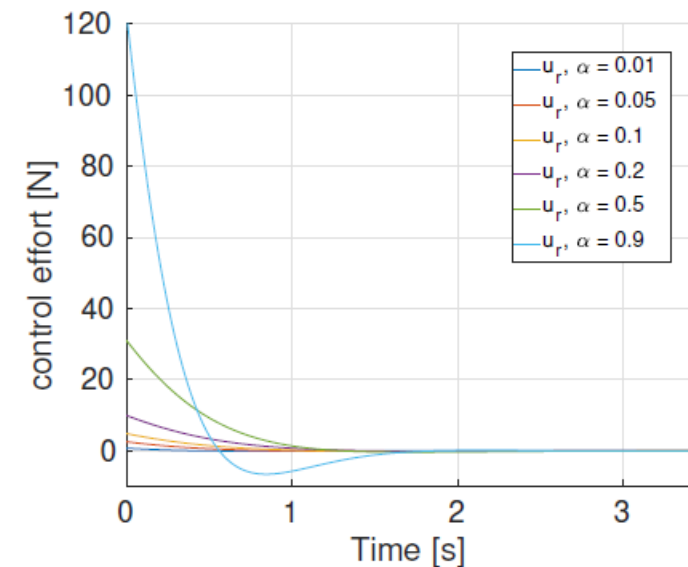
Simulations - $Q_{r,r} = \text{diag}(0.1, 0.001)$



(a) position state history with increasing α

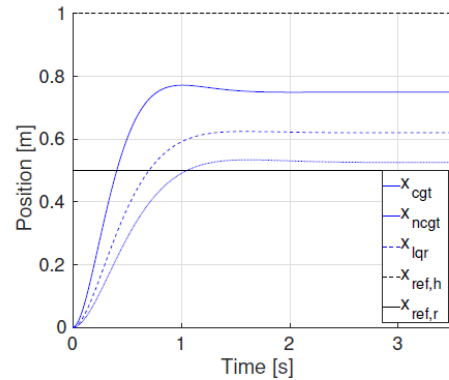


(b) human control action with increasing α .

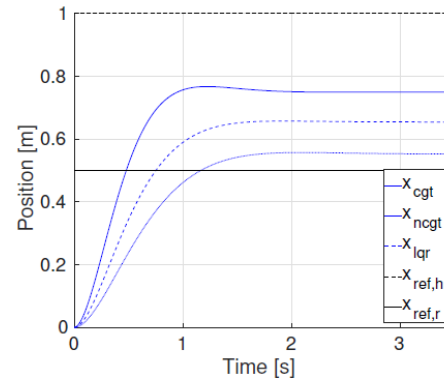


(c) robot control action with increasing α .

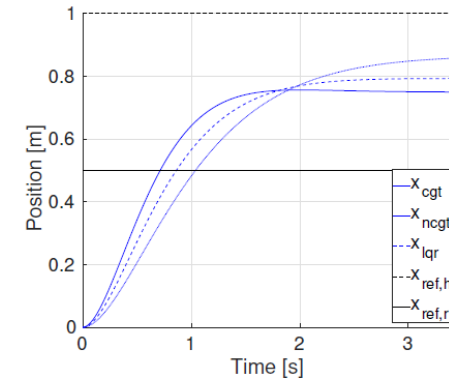
Simulations - $R_{r,r} = \{5e^{-5}, 1e^{-4}, 1e^{-3}\}$



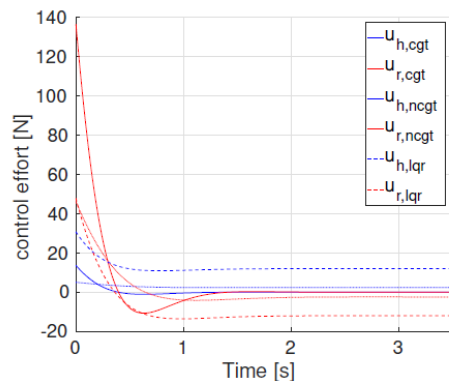
(a) state with $R_r = 0.00005$.



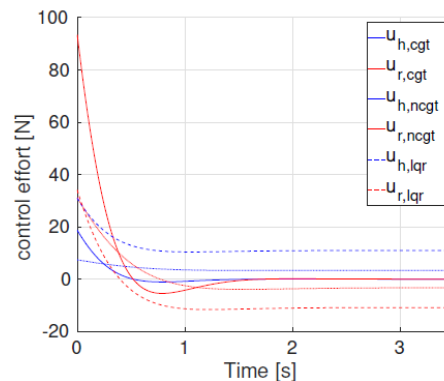
(b) state with $R_r = 0.0001$.



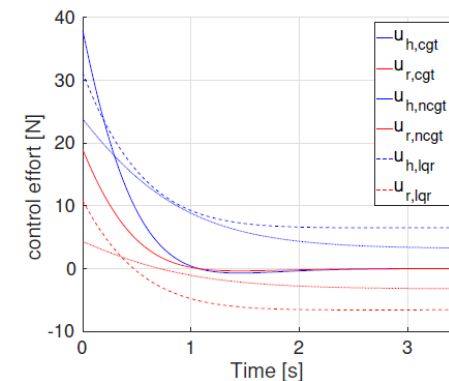
(c) state with $R_r = 0.001$.



(d) control actions with $R_r = 0.00005$.



(e) control actions with $R_r = 0.0001$.



(f) control actions with $R_r = 0.001$.

Simulations – cost

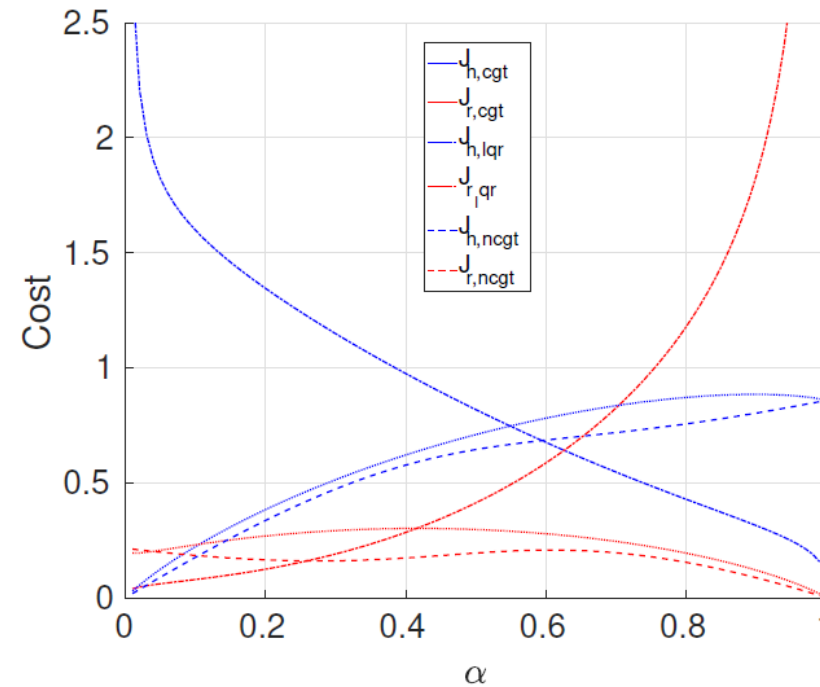
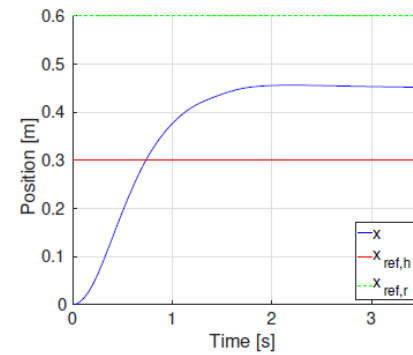
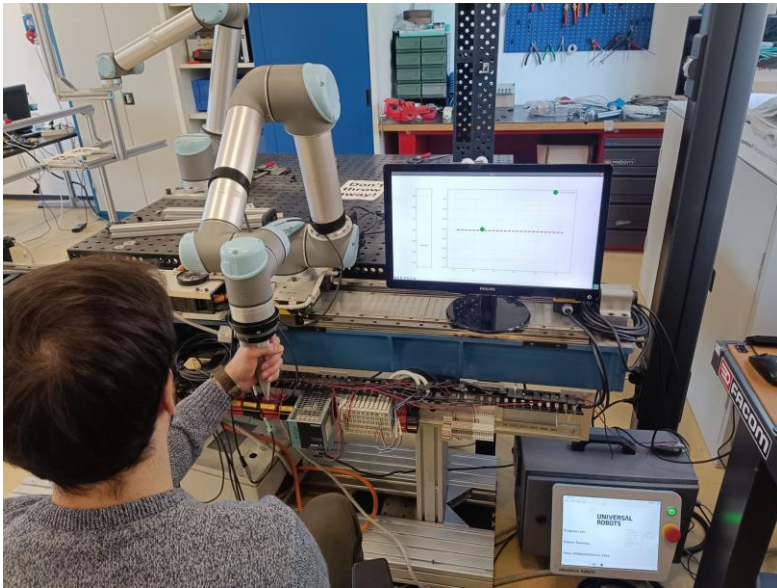
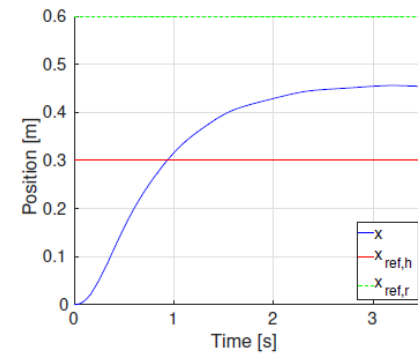


Fig. 4: Human (blue lines) and robot (red lines) costs for the three controllers for various α in the simulated cases.

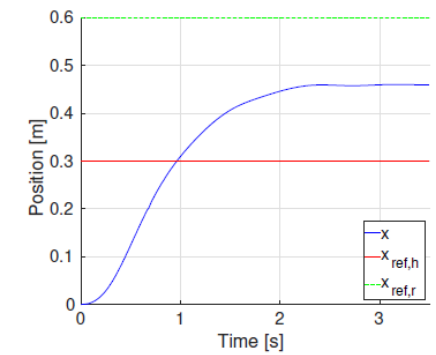
Experimental results



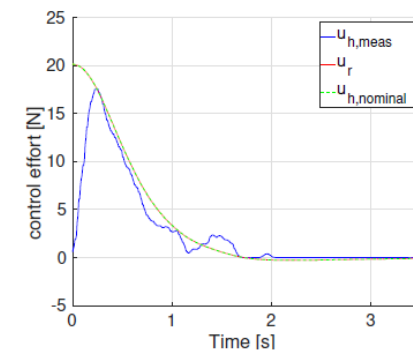
(a) measured position for CG with $\alpha = 0.5$.



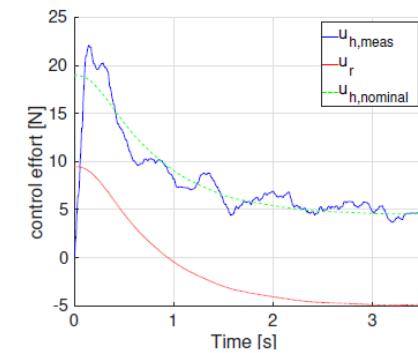
(b) measured position for LQR with $\alpha = 0.5$.



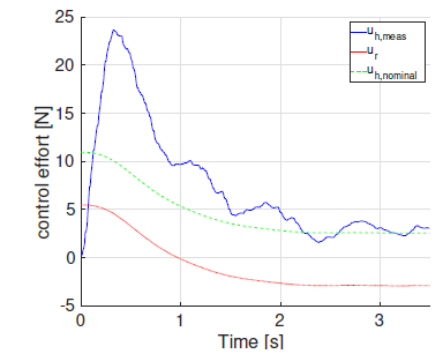
(c) measured position for NCG with $\alpha = 0.5$.



(d) CG measured human and robot control efforts and nominal human control effort.



(e) LQR measured human and robot control efforts and nominal human control effort.



(f) NCG measured human and robot control efforts and nominal human control effort.

Experimental results

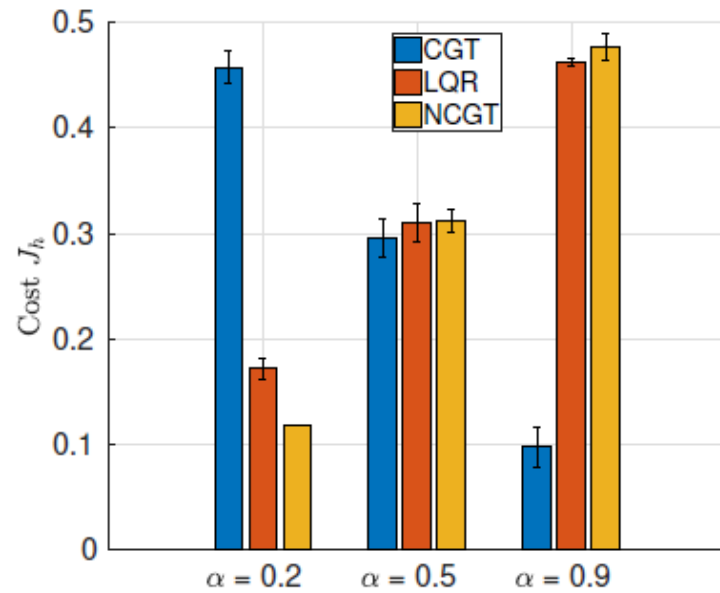


Fig. 6: Computed Human cost varying α .

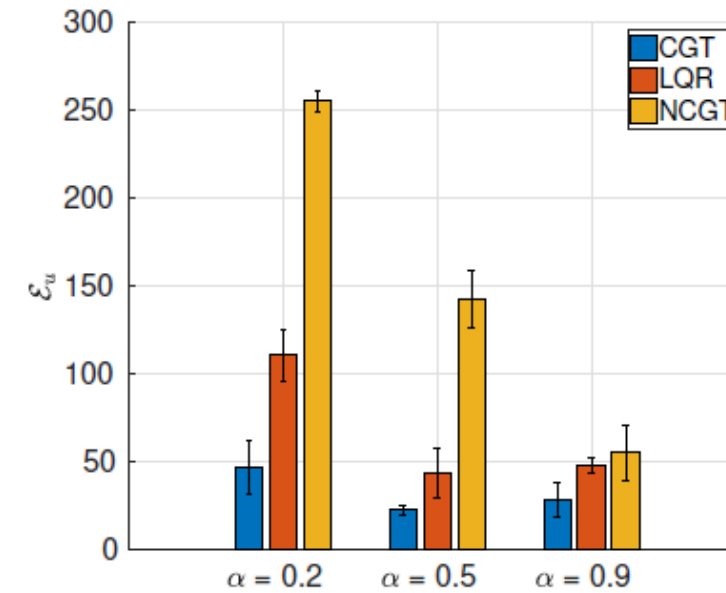


Fig. 7: human's control errors for the three controllers varying α .

Conclusions

Cooperative GT:

- i) high values of α : the robot tends to assist the human in pursuing his/her goal. This situation is seek when the human is leading the task and wants the robot's assistance as follower
- ii) low values of α : the robot expects the human assistance, this situation does not apply to real-world applications

Non-Cooperative GT/LQR:

- i) Low values of α : the robot tends to pursue its own goal. This situation is desirable for some specific sub-tasks (e.g., precise positioning of a large co-manipulated object)

Target – Role Arbitration¹

Role Arbitration: dynamically switch the leader-follower role during a task

Implementation: dynamically switch the Cooperative Non-Cooperative formulation of the problem

¹P. Franceschi, N. Pedrocchi and M. Beschi, "Human–Robot Role Arbitration via Differential Game Theory," in *IEEE Transactions on Automation Science and Engineering*, doi: 10.1109/TASE.2023.3320708

Role Arbitration-application





Thank you for your attention!
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